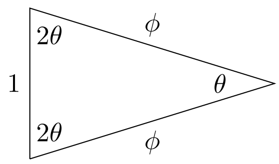
4601. To differentiate the function $x \mapsto \sec x$ from first principles, the following limit is set up:

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\sec(x+h) - \sec x}{h}.$$

Evaluate this limit algebraically to prove that

$$\frac{d}{dx} \sec x = \tan x \sec x.$$

4602. A *golden triangle* has angles in the ratio $1 : 2 : 2$.



Show that a golden triangle has lengths in the ratio $1 : \phi : \phi$, where ϕ is the golden ratio

$$\phi = \frac{\sqrt{5}+1}{2}.$$

You may use the fact that $\cos \frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$.

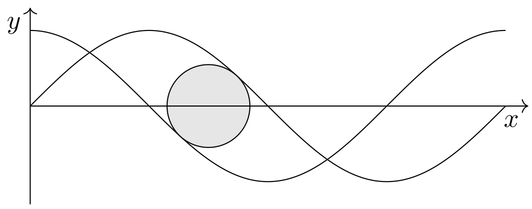
4603. A curve has x intercept 1 and derivative

$$\frac{dy}{dx} = 1 - y^2.$$

Show that, for points on the curve,

$$y = \frac{e^{2x} - e^2}{e^{2x} + e^2}.$$

4604. The diagram shows the curves $y = \sin x$ and $y = \cos x$, and a circle, centred on the x axis, that is tangent to both.



(a) Explain why the centre of the circle is $\left(\frac{3\pi}{4}, 0\right)$.

(b) Show that the equation of the normal to the curve $y = \sin x$ at the point $(a, \sin a)$ is

$$y = (a - x) \sec a + \sin a.$$

(c) Show that, for such a normal to be a radius of the circle, a must satisfy the equation

$$\sin 2a + 2a - \tfrac{3}{2}\pi = 0.$$

(d) Use a numerical method to find a to 5sf.

(e) Hence, show that the diameter of the circle is approximately 1.095.

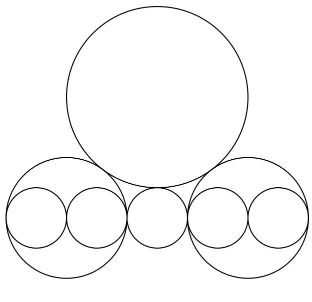
4605. Prove the *inclusion-exclusion principle* for three sets, which states that, for events A, B, C,

$$\begin{aligned} &\mathbb{P}(A \cup B \cup C) \\ \equiv\ &\mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) \\ &\quad - \mathbb{P}(A \cap B) - \mathbb{P}(B \cap C) - \mathbb{P}(C \cap A) \\ &\quad\quad + \mathbb{P}(A \cap B \cap C). \end{aligned}$$

4606. Sketch $y = \dfrac{1}{x^4 - 2x^2a^2 + a^4}$, where $a \neq 0$.

4607. A tangent is drawn to the curve $y = x^k - kx^{k+1}$ at $x = 1$. This tangent crosses the x axis at $x = 3$. Determine all possible values of the constant k.

4608. In the diagram below, the five smallest circles have unit radius, and their centres are collinear.



Determine the radius of the largest circle.

4609. The SHM equation is defined as $\ddot{x} = -\omega^2 x$, where ω is a constant and $\ddot{x}$ signifies the second time derivative of x. The variable x represents position relative to an origin.

(a) Show that, if the SHM equation is satisfied, then resultant force is proportional to distance from the origin, and directed towards it.

(b) A solution curve $x = A \sin(\omega t + \varepsilon)$ is proposed, in which $A > 0$ and $\varepsilon \in [0, 2\pi)$ are constants.

i. Verify that this solution curve satisfies the SHM equation.

ii. The motion starts at $x = -2$, and the range of positions is $\{x \in \mathbb{R} : -4 \leq x \leq 4\}$. Find all possible values of A and ε.

4610. Show that $\displaystyle\int_0^{\frac{\pi}{2}} e^{2x} \sin 4x \, dx = \tfrac{1}{5}(1 - e^{\pi})$.

4611. An ellipse has equation $\frac{1}{9}x^2 + \frac{1}{16}y^2 = 1$.

(a) Give parametric equations for this curve in the form $x = a\cos\theta$, $y = b\sin\theta$, for $a, b > 0$.

(b) A normal is drawn to the ellipse at the point with parameter θ. Show that this normal has an x intercept given by

$$x = -\tfrac{7}{3}\cos\theta.$$

4612. Factorise $144x^4 - 468x^3 - 142x^2 + 52x + 14$.

4613. Consider the curve $y = x^7 - 14x^4 + 49x$.

(a) Show that this curve has exactly two SPs, one of which is a minimum and one of which is a maximum.

(b) Show that y changes sign exactly once.

4614. The area of a circle of radius r is given by

$$A = 4\int_0^r \sqrt{r^2 - x^2}\,dx.$$

(a) Explain the significance of the limits 0 to r and the factor of 4.

(b) By carrying out the integral, prove that the area of a circle is πr^2.

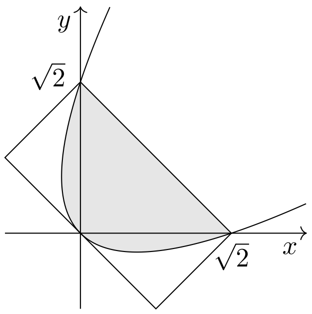
4615. A random number generator produces X values with probabilities given by $X \sim B(10, 0.45)$. Find

$$\mathbb{P}\big(X \in [2,5) \mid X \in [3,6)\big).$$

4616. Curve C, shown below, is given by the equation

$$x + y = \tfrac{1}{\sqrt{2}}(x - y)^2.$$

A rectangle has been added to the diagram.



(a) Show that C can be written $b = a^2$, where

$$a = \tfrac{1}{\sqrt{2}}(x - y), \quad b = \tfrac{1}{\sqrt{2}}(x + y).$$

(b) Find the values of a at the intersections of the rectangle and C.

(c) Hence, show that the shaded area is given by

$$A = k - \int_{-1}^{1} a^2\,da,$$

where k is to be determined.

(d) Find the area of the shaded region.

4617. You are given that the coordinates (x, y) of the position of a particle satisfy

$$2x\cot y = \frac{dy}{dx}(x^2 + 1).$$

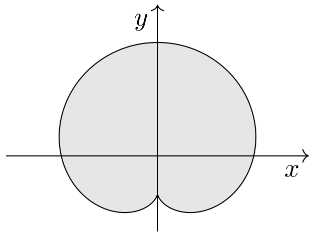
The particle passes through the (x, y) origin. Show that, in Cartesian form, its path is

$$(x^2 + 1)\cos y = 1.$$

4618. The cardioid below has parametric equations

$$x = \tfrac{1}{2}\sin 2t + \sin t,$$
$$y = \tfrac{1}{2}\cos 2t + \cos t.$$

The curve encloses a region of area A:



Show that $A = \frac{3\pi}{2}$.

4619. A rational function is defined as

$$f(x) = \frac{4x + 1}{2x^2 + 1}.$$

Sketch the curve $y = f(x)$, finding the coordinates of any axis intercepts and stationary points.

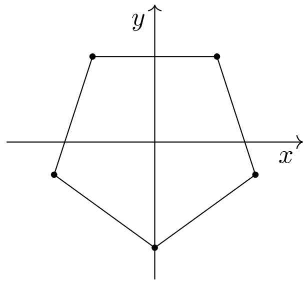
4620. A computer assigns the function f as sin or $\sin^2$ with equal probability. Then, it chooses a value for x from the domain $[0, 2\pi)$. Its output is $f(x)$. Given that $f(x) > \frac{1}{2}$, find the probability that the computer assigned sin.

4621. Show that the graph $y = \displaystyle\sum_{r=1}^{n} \frac{1}{x - r}$ has

(a) $n + 1$ asymptotes,

(b) $n - 1$ roots.

4622. A regular pentagon of unit side length is drawn in an (x, y) plane, centred on the origin.



Its edges are labelled $E_1, ..., E_5$ clockwise. Edge E_1 is described by the parametric vector equation $\mathbf{r} = t\mathbf{i} + \mathbf{j}$, for $t \in [-a, a]$.

(a) Show that $a = \tan 36°$.

(b) Find the acute angle between E_2 and $\mathbf{i}$.

(c) Hence, show that the equation of E_2 may be expressed as

$$\mathbf{r} = (\tan 36° + t\cos 72°)\mathbf{i} + (1 - t\sin 72°)\mathbf{j}.$$

(d) Show that, in the above, $t \in [0, 2\tan 36°]$.

4623. A monic quartic curve has equation

$$y = x^4 + ax^3 + bx^2 + cx + d.$$

This curve has a stationary point at $(3, -11)$, is convex on the set $(-\infty, 0)$, concave on the set $(0, 4)$ and convex on the set $(4, \infty)$.

Determine a, b, c, d.

4624. This question concerns the definite integral

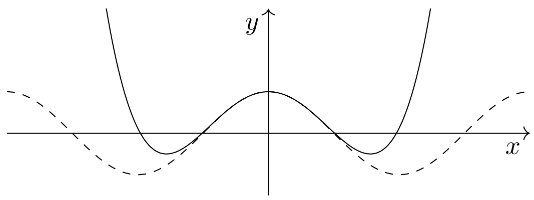
$$I = \int_0^1 (\ln x)^2 \, dx.$$

(a) Using the substitution $u = \ln x$, show that

$$I = \lim_{k \to -\infty} \int_k^0 u^2 e^u \, du.$$

(b) Hence, determine the value of I.

4625. The cosine function, for fairly small angles, is to be approximated with a quartic by matching the values of its zeroth, first, second, third and fourth derivatives at $\theta = 0$.



(a) Write down the standard quadratic small-angle approximation to $\cos\theta$.

(b) Show that the coefficient of θ^3 is zero.

(c) Determine the coefficient of θ^4.

4626. Simultaneous equations in x and y are given as

$$|y| = x + 2,$$
$$|x| = y + 2.$$

Find all (x, y) points that satisfy both, expressing your answer algebraically in set notation.

You may wish to use the notation $\mathbb{R}^2$ to represent the set of all real (x, y) pairs.

4627. In this question, use the following approximations, valid for small angles x in radians:

$$\sin x \approx x - \tfrac{1}{6}x^3,$$
$$\cos x \approx 1 - \tfrac{1}{2}x^2.$$

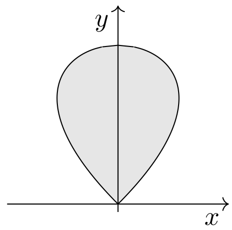
Show that, for small x, $\tan x \approx x + \frac{1}{3}x^3$.

4628. In a model, it is proposed that three variables $x, y, z \geq 0$ are related as follows:

$$x + y = z, \quad \frac{dy}{dx} = 2y, \quad \frac{dz}{dx} = 3y^2.$$

Show that this model is not well defined.

4629. The shaded region in the diagram below is defined by the inequality $x^2 + y^3 - y^2 \leq 0$.



Determine the exact area of the shaded region.

4630. You are given that $(x^2 + k)(x^2 + 4x + k) = 0$ has exactly one real root. Determine the value of k.

4631. Curve C is defined by the following equation:

$$y = \frac{1}{x+1}.$$

Point A on curve C is at $\left(a - 1, \frac{1}{a}\right)$.

(a) Show that the equation of the tangent at A is

$$y = -\frac{1}{a^2}x + \frac{2a-1}{a^2}.$$

(b) Hence, or otherwise, find the equation of the tangent to C that passes through the origin.

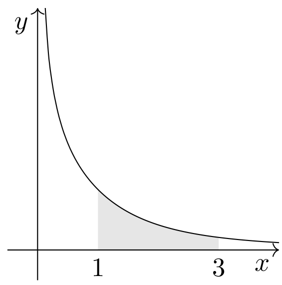
4632. A hypothesis test is enacted to determine whether a random number generator is working correctly. The generator should produce values uniformly and continuously distributed across the interval $\{x : 0 \leq x \leq 1\}$. A sub-interval $[0.231, 0.629]$ is chosen at random by another process. A sample of 100 values is then generated.

(a) Show that the expected number of values in the sub-interval is 39.8.

(b) In fact, 55 values are observed to lie in the sub-interval. Conduct a hypothesis test at the 1% significance level.

4633. The diagram shows a region formed by the lines $y = 0$, $x = 1$ and $x = 3$ and the curve

$$xy^2 + x^2y - 2 = 0.$$



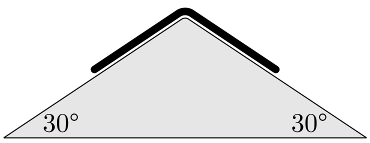
Find, to 4sf, the area of the shaded region.

4634. You are given that the following equations hold:

$$-6 + 2x - 3y + xy = 0,$$
$$x^2y^2 + 9y^2 - 4x^2 - 36 = 0.$$

Find all possible values of x and y.

4635. A uniform, inextensible chain two metres long is placed in equilibrium on a smooth surface in the shape of a roof, whose cross-section is as shown. The apex of the roof is a small circular arc whose radius can be neglected.



The chain is now set in motion, remaining in the plane depicted. This is modelled as follows: at time $t = 0$, the entire chain begins to slide, from the symmetrical position shown, at an initial speed u ms^{-1}.

(a) Show that the acceleration is given by

$$\frac{d^2x}{dt^2} = \begin{cases} \frac{1}{2}gx & 0 \le x < 1, \\ \frac{1}{2}g & x \ge 1. \end{cases}$$

(b) Sketch an acceleration-time graph.

(c) Verify that, for $0 \le x < 1$, the displacement is

$$x = \frac{u}{k}e^{kt}.$$

The constant k is to be determined.

4636. In this question, the derivative with respect to time is expressed using Newton's dot notation.

The radius r of a circle is changing such that

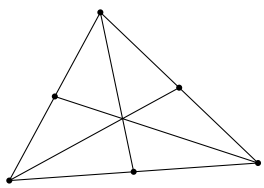
$$\dot{r} \times r^2 = 1.$$

Show that the rate of change of area satisfies

$$\dot{A} \times \sqrt{A} = k,$$

where k is a constant to be determined.

4637. A *median* of a triangle is a straight line joining a vertex to the midpoint of the opposite side.



Prove that the three medians of a triangle ABC are concurrent.

4638. Determine the exact value of $\int_0^1 4^x - 2^x \, dx$.

4639. A particle moving in a circle at constant speed is represented by the position vector

$$\mathbf{r} = r\cos\omega t\,\mathbf{i} + r\sin\omega t\,\mathbf{j},$$

where r and ω are constants.

(a) Find the velocity vector $\mathbf{v}$ and show that it is always perpendicular to the position vector.

(b) Show that the speed v is given by $v = r\omega$.

(c) Find the acceleration vector $\mathbf{a}$.

(d) Hence, derive the formula for the magnitude of the centripetal acceleration in circular motion:

$$a = \frac{v^2}{r}.$$

4640. Show that $\int_{-\pi}^{\pi} \sin^2 x \cos^2 x \, dx = \frac{1}{4}\pi$.

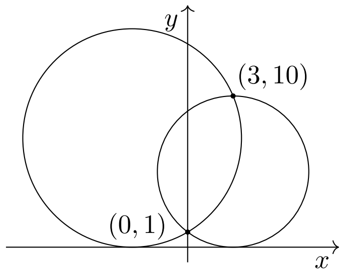
4641. True or false?

(a) Every quartic has a cubic factor,

(b) Every quintic has a quartic factor,

(c) Every sextic has a quintic factor.

4642. The diagram shows two circles which pass through $(0, 1)$ and $(3, 10)$, and are tangent to the x axis.



Find the exact radii of the circles.

4643. Prove the following identity:

$$\cos^4\theta \equiv \frac{3 + 4\cos 2\theta + \cos 4\theta}{8}.$$

4644. A function f has instruction

$$\mathrm{f}(x) = \frac{20x^2}{(2x-9)^2}.$$

(a) Explain what is meant by

i. "$y = \mathrm{f}(x)$ has a double root."

ii. "$y = \mathrm{f}(x)$ has a double asymptote."

(b) Show that, as $x \to \pm\infty$, $y \to 5$.

(c) Sketch the graph $y = \mathrm{f}(x)$, marking any roots and asymptotes.

4645. A differential equation is given, together with the boundary condition that $y = \frac{1}{2}$ when $x = 0$, as

$$\frac{dy}{dx}\sec x = y^2 - y.$$

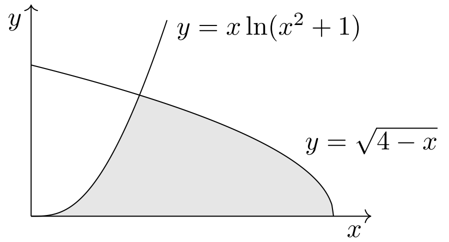
Find y as a function of x.

4646. A curve is given by

$$y^3 + xy = 2y + 4x - 10.$$

The line $4x + 3y = k$ is normal to the curve at a point with integer coordinates. Find k.

4647. Evaluate, to 3 significant figures, the area of the shaded region in the diagram.



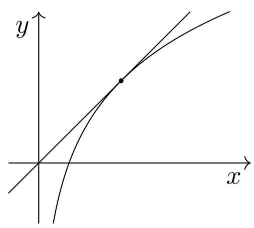
4648. Explain which, if any, of the implication symbols $\Longrightarrow$, $\Longleftarrow$, $\Longleftrightarrow$ may be used to link the following statements, for polynomial functions f and g:

① $\text{f}(x) - \text{g}(x)$ has a factor of $(x-a)^2$,
② $\text{f}'(a) = \text{g}'(a) = 0$.

4649. Three red, three green and three blue marbles are placed in a bag. Three marbles are then drawn from the bag, without replacement.

Determine the probability of getting three marbles of different colours.

4650. The curve $y = \log_k x$, for some constant k, has the line $y = x$ as a tangent.



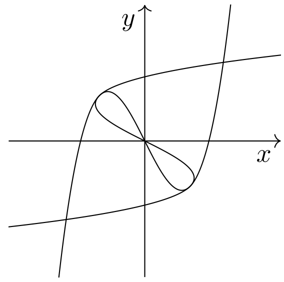
Find the coordinates of the point of tangency.

4651. Integral I is given, in terms of a constant a, as

$$I = \int_{-a}^{5-a} \frac{1}{(t+a+1)(t+a+2)(t+a+3)}\,dt.$$

By using the substitution $t + a = x$, or otherwise, determine the value of I. Give your answer in the form $\ln k$, for $k \in \mathbb{Q}$.

4652. The curves $y = x^3 - 2x$ and $x = y^3 - 2y$ are shown.



Find all points of intersection.

4653. A function S is defined as

$$\text{S}(k) = \tfrac{1}{6}k(k+1)(2k+1).$$

(a) Verify that $\text{S}(1) = 1$.

(b) Prove algebraically that

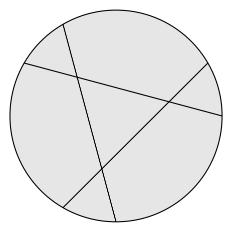
$$\text{S}(k) + (k+1)^2 \equiv \text{S}(k+1).$$

(c) Hence, explain why $\text{S}(n)$ gives the sum of the first n squared integers.

4654. Prove the *sum-to-product formula*

$$\sin A + \sin B \equiv 2\sin\left(\tfrac{A+B}{2}\right)\cos\left(\tfrac{A-B}{2}\right).$$

4655. The *lazy caterer's sequence* is $\{P_n\}$, where P_n is the largest number of pieces into which a circular cake can be cut using n straight line cuts. In the example shown, $P_3 = 7$.



Prove that $P_n = \dfrac{n^2+n+2}{2}$.

4656. Two parabolae are drawn, with equations

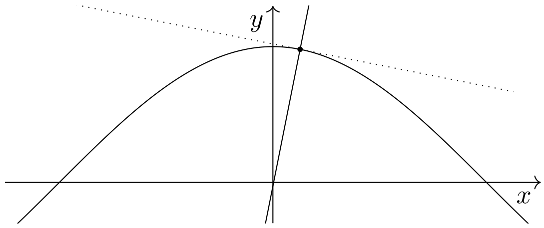
$$y = x^2,$$
$$y = (x-2)^2.$$

On each, a tangent is then drawn, which passes through the point $(1, -5/4)$ and does not intersect the other parabola. Show that these two tangents meet at right angles.

4657. Show that $\displaystyle\sum_{r=1}^{\infty} \frac{4}{r^2+2r} = 3$.

4658. Prove that $\cos(\arctan x) \equiv \dfrac{1}{\sqrt{x^2+1}}$.

4659. Show that, for small angles θ, the normal to the curve $y = \cos x$ at $x = \theta$ crosses the x axis at a negligible distance from the origin.



4660. The curve $y = x^p + x^{p-1}$ is drawn, where $p > 1$ is a constant. This curve is transformed by stretches parallel to the x and y axes, with scale factors c and d respectively. The equation of the image is

$$y = ax^p + bx^{p-1}.$$

Show that $cd = \dfrac{b^{p+1}}{a^p}$.

4661. The *Riemann zeta function* is defined by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

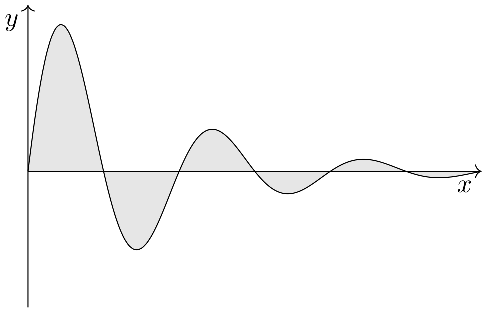
A proof that the sum is finite for $s = 2$ begins with

$$\zeta(2) < 1 + \sum_{n=2}^{\infty} \frac{1}{n(n-1)}.$$

(a) Justify this first step.

(b) Complete the proof.

4662. The curve $y = 2e^{-x} \sin x$ is shown below, for $x \geq 0$. The regions enclosed by the curve and the x axis are shaded.



Show that, as the pattern continues for $x \to \infty$, the total shaded area tends to

$$A = \frac{e^\pi + 1}{e^\pi - 1}.$$

4663. Two quadratic equations are given as

$$x^2 + 2px + q = 0,$$
$$x^2 + qx + p^2 = 1.$$

You are told that each equation has exactly one real root. Find all possible values of the constants p and q.

4664. A sequence is defined, for some starting value, by

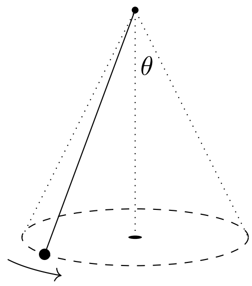
$$u_{n+2} = u_{n+1} - u_n.$$

Prove that the sequence either has period 6 or is constant at value 0.

4665. For a particle moving at constant speed v around a circle of radius r, the centripetal acceleration is directed towards the centre of the circle, and has magnitude

$$a = \frac{v^2}{r}.$$

A *conical pendulum* is formed of a small bob of mass m attached to one end of a string of length l, the other end of which is fixed. The bob moves in a horizontal circle, with the string at a constant angle θ to the vertical.



Prove that the time taken for one circuit is

$$t = 2\pi\sqrt{\frac{l\cos\theta}{g}}.$$

4666. Variable X has distribution $B(n, 1/5)$. For some n,

$$\mathbb{P}\big(X = 3\big) = \mathbb{P}\big(X = 4\big).$$

Using an algebraic method, determine n.

4667. This question is about different orderings of the composition of functions. For functions f and g, a new function $\text{f} \odot \text{g}$ is defined as

$$\text{f} \odot \text{g}(x) = \text{fg}(x) - \text{gf}(x).$$

(a) Show that, when $\text{f}(x) = x^2$ and $\text{g}(x) = 2x + 1$, $\text{f} \odot \text{g}(x) = 2x^2 + 4x$.

(b) Let $\text{f}(x) = \sin x$, $\text{g}(x) = 2x$. There is exactly one value of $x \in (-\pi, \pi)$ for which

$$\text{f} \odot \text{g}(x) = \tfrac{3\sqrt{3}}{2}.$$

Find this value, giving your answer exactly.

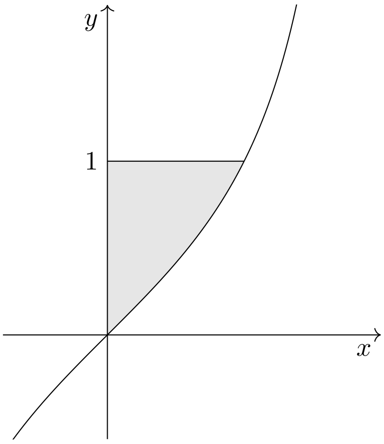
(c) For particular functions f and g,

$$\text{f} \odot \text{g}(x) \equiv 0.$$

Explain what this signifies about f and g.

4668. Prove that $\sqrt{p}+\sqrt{q}$, for primes p, q, is irrational. You may assume that, if k isn't a square number, then $\sqrt{k}$ is irrational.

4669. Show that the area enclosed by the curve $y=\tan x$ and the lines $x=0$ and $y=1$ is $\frac{1}{4}(\pi-\ln 4)$.



4670. Sketch $y=-xe^{x+1}$, marking the coordinates of any stationary points.

4671. A set of three different letters is chosen at random from the alphabet. Find the probability that the set consists of three letters that are consecutive alphabetically.

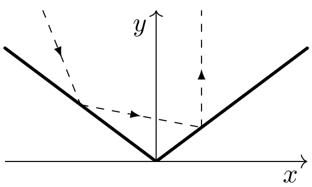
4672. A sequence has starting value x_1. It is thereafter defined by the following iteration, in which a and b are constants with $a \neq 1$:

$$x_{n+1}=ax_n+b.$$

Prove that the nth term is given by

$$x_n=a^{n-1}x_1+\frac{a^{n-1}-1}{a-1}b.$$

4673. A solar reflector is being designed. It is modelled with the shape $y=k|x|$. The incoming light makes an angle θ with the vertical, and the purpose of the design is to reflect light vertically, as shown.



Show that k must be set to $\tan\frac{1}{4}(\pi-\theta)$.

4674. A quartic graph is given by the equation

$$y=x^4-2x^2+2x+1.$$

Show that, for a constant $k\in\mathbb{N}$ to be determined, the line $y=kx$ is tangent to the curve at two distinct points.

4675. A differential equation linking y and x is given as

$$\frac{dy}{dx}-4\frac{dx}{dy}=3.$$

Show that any solution curve is of one of the two forms $y=4x+k_1$ or $y=-x+k_2$.

4676. The graph $y=x^2$ has a normal drawn to it at (a,a^2), which intersects the curve again at $y=\frac{25}{8}$. Find all possible values of a.

4677. Two simultaneous equations in x and y are defined, with coefficients in terms of constants p and q, as

$$(p-1)x+(q-1)y=0,$$
$$p^2x+q^2y=1.$$

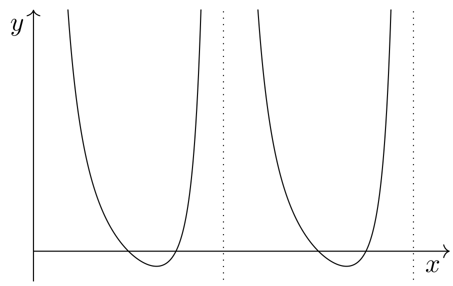
You are given that these equations do not have a unique (x,y) solution point. Sketch, on a set of (p,q) axes, the locus of possible values of p and q.

4678. Show that $\displaystyle\int_1^2 \frac{x^3+2x^2+4x+4}{x^3+2x^2}\,dx=2+\ln\frac{3}{2}$.

4679. Over the largest possible domain, f is defined as

$$\text{f}(x)=\text{cosec}^2\,x+\cot x-1.$$

The graph below is $y=\text{f}(x)$, for $x\in[0,2\pi)$:



Solve $\text{f}(x)\geq 0$, for $x\in[0,2\pi)$.

4680. To differentiate $y=\sin x$ from first principles, the following limit is set up

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{\sin(x+h)-\sin(x-h)}{2h}.$$

Evaluate the limit to prove that $\dfrac{dy}{dx}=\cos x$.

4681. The formula for the arc length of a curve $y=\text{f}(x)$ between $x=a$ and $x=b$ is

$$S=\int_a^b\sqrt{1+\big(\text{f}'(x)\big)^2}\,dx.$$

Determine the arc length S of the curve $y=\frac{2}{3}x^{\frac{3}{2}}$ between $x=0$ and $x=48$.

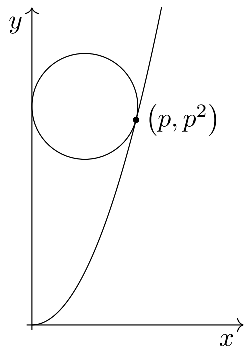
4682. Prove that the largest cone that will fit inside a sphere has $8/27$ of its volume.

4683. A function is defined over $(-1, \infty)$ by

$$\mathrm{f}(x) = \frac{1}{(x+1)\ln(x+1)}.$$

Find all functions F whose derivative is f.

4684. A unit circle is tangent to the positive y axis and to the curve $y = x^2$ at (p, p^2).



Find p, giving your answer to 4sf.

4685. Solve the following simultaneous equations:

$$x^3 - y^3 = \tfrac{7}{16},$$
$$x - y = 1.$$

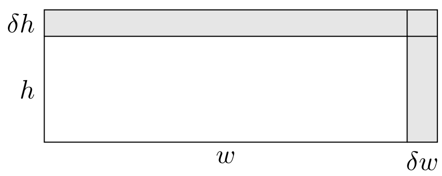
4686. Show that $\displaystyle\int_0^{\pi} \sin^4 x \, dx = \tfrac{3\pi}{8}$.

4687. The locus of the following equation, in which k is a positive constant, is a polygon:

$$\left|\sqrt{3}x + y\right| + \left|\sqrt{3}x - y\right| + \left|2y\right| = k.$$

Name the polygon.

4688. A rectangle has variable width w and height h. In a short period of time δt, w and h increase by δw and δh respectively, thus increasing the area of the rectangle by δA (shaded).



(a) Show that $\dfrac{\delta A}{\delta t} = h\dfrac{\delta w}{\delta t} + w\dfrac{\delta h}{\delta t} + \dfrac{\delta w \delta h}{\delta t}$.

(b) Explain why, as δt tends to zero, the last term also tends to zero.

(c) Hence, prove the product rule.

4689. Prove that, for any positive constant a,

$$\int_{-a}^{a} \frac{1}{\sqrt{a^2 - x^2}} \, dx = \pi.$$

4690. On an set of Cartesian axes, shade the region(s) that satisfy the inequality $(y - x^2)(x - y^2) \geq 0$.

4691. A projectile is launched from ground level. The Cartesian equation of its trajectory is

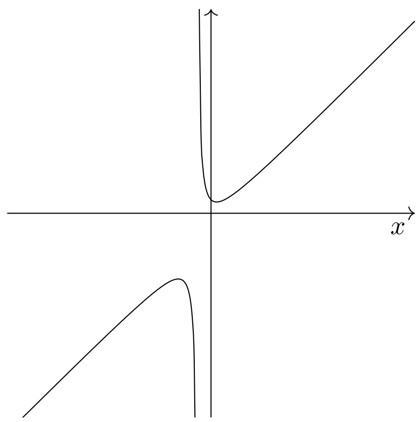
$$y = x - \frac{gx^2}{32}.$$

Find the speed of the projectile at its highest point.

4692. Find the range of $\arctan(\cos x)$, for $x \in \mathbb{R}$.

4693. A student is trying to find the oblique asymptote of the graph

$$y = \frac{x^2 + 1}{x + 1}.$$



He writes: "As $x \to \infty$, the 1's become negligible, so the curve tends to $y = x^2/x$, i.e $y = x$." Explain the error in this argument, and find the correct equation of the oblique asymptote.

4694. Show that $\displaystyle\lim_{x \to a} \frac{x^2 - 3a + 3x - ax}{x^2 - a + x - ax} = 1 + \frac{2}{a+1}$.

4695. Show that $\dfrac{d}{dx}(\cos 2x \tan 2x) = 2\cos 2x$.

4696. A curve is defined, for $0 \leq t < 2\pi$, by

$$x = 3\cos 2t,$$
$$y = 6\sin 2t.$$

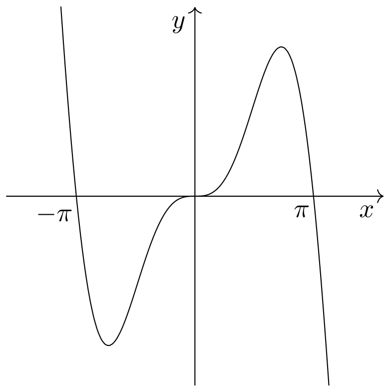
Show that $\dfrac{d^2y}{dx^2} = -\dfrac{144}{y^3}$.

4697. Prove that there can never be events X and Y for which the following inequalities all hold:

$$\mathbb{P}(X) > \tfrac{1}{2},$$
$$\mathbb{P}(X' \cup Y) > \tfrac{3}{4},$$
$$\mathbb{P}(Y' \mid X) > \tfrac{1}{2}.$$

4698. "$(x + y)^2 + (x - y)^2 = 2$ defines a circle."
True or false?

4699. A student sketches the following graph, claiming it to be $y = x(1 - \cos 2x)$. Show that this is incorrect.



4700. A curve is given, in terms of a parameter $t \in \mathbb{R}$, by the equations

$$x = \frac{2t}{1+t^2}, \qquad y = \frac{1-t^2}{1+t^2}.$$

(a) Verify that $P : \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lies on the curve.

(b) Find the equation of the tangent at P.

——— End of 47th Hundred ———